Natural dark matter from type I string theory

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Abstract: We study neutralino dark matter within a semi-realistic type I string model, where supersymmetry breaking arises from F-terms of moduli fields parameterised in terms of Goldstino angles, which automatically gives rise to non-universal soft third sfamily and gaugino masses. We study the fine-tuning sensitivities for dark matter and electroweak symmetry breaking across the parameter space of the type I string model, and compare the results to a similar analysis in the non-universal MSSM. Within the type I string model we find that neutralino dark matter can be naturally implemented in the $\tilde{\tau}$ bulk region, the Z^0 resonance region and the maximally tempered Bino/Wino/Higgsino region, in agreement with the results of the non-universal MSSM analysis. We also find that in the type I string model the "well-tempered" Bino/Wino region is less fine-tuned than in the MSSM, whereas the $\tilde{\tau}$ co-annihilation region exhibits a significantly higher degree of fine-tuning than in the MSSM.

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1. Introduction

The prediction of neutralino dark matter is generally regarded as one of the successes of the Minimal Supersymmetric Standard Model (MSSM). However the successful regions of parameter space allowed by WMAP and collider constraints are quite restricted. In a recent paper [1] we discussed fine-tuning with respect to both dark matter and Electroweak Symmetry Breaking (EWSB) and explored regions of MSSM parameter space with non-universal gaugino and third family scalar masses in which neutralino dark matter may be implemented naturally. For example, we found that the recently proposed "well tempered neutralino" regions [2] involve substantial fine-tuning of MSSM parameters in order to satisfy the dark matter constraints, although the fine tuning may be ameliorated if several annihilation channels act simultaneously. To overcome this we proposed the "maximally tempered neutralino" comprising substantial components of Bino, Wino and Higgsino [1], and showed that it leads to a reduction in fine-tuning. Moreover we also found other regions of parameter space which were not "well tempered" that exhibit low dark matter fine tuning. For example the $\tilde{\tau}$ co-annihilation region was shown to have low fine-tuning, while the bulk region consisting of t-channel slepton exchange (achievable with non-universal gaugino masses) was shown to involve no dark matter tuning at all corresponding to "supernatural dark matter". In all cases the usual MSSM fine tuning associated with EWSB remained.

Though such a non-universal MSSM provides a general framework for studying natural dark matter regions, it may not be realistic to regard the mass terms in the soft supersymmetry (SUSY) breaking Lagrangian as fundamental inputs since the soft masses merely

parameterise the unknown physics of SUSY breaking. In any realistic model of SUSY breaking the soft breaking terms in the Lagrangian should be generated dynamically. It is the parameters that define the mechanism of SUSY breaking that should be taken as the fundamental inputs. This immediately raises a difficulty as the true origin of SUSY breaking is unknown. In string theory the unknown SUSY breaking dynamics may be manifested as F-term vacuum expectation values (VEVs) of hidden sector moduli fields appearing in the theory. Therefore the values of these F-terms may be regarded as being more fundamental input parameters than the soft mass terms of the MSSM. Although the values of the F-terms are unknown, they may be parameterised in terms of so called Goldstino angles which describe the relative magnitude of the F-terms associated with the different moduli fields, as was done for example in type I string theories in [3]. A more reliable estimate of fine-tuning sensitivity should therefore result from using such Goldstino angles, together with the gravitino mass $m_{3/2}$, and some other undetermined electroweak parameters such as the μ parameter and the ratio of Higgs vacuum expectation values $\tan\beta$ as inputs. Therefore fine-tuning should more properly be calculated with respect to these inputs. It is possible that fine-tuning when calculated in terms of such inputs could yield very different results.

In this paper we extend our previous analysis of the non-universal MSSM to a semi-realistic type I string theory model of the form originally proposed in [4] and phenomenologically analysed in [5] (see also [6–9]). Using such a string model we can address two questions. Firstly, how does the fine-tuning of a particular dark matter region in the non-universal MSSM [1] compare to a similar region in the string model? Secondly, do some regions of SUSY breaking parameter space in the string model more naturally explain dark matter and electroweak symmetry breaking than others? The model we use to address these points is the type I string inspired model in [4] in which we can obtain SUSY breaking from any of twisted (Y) moduli, untwisted (T) moduli or the dilaton (S). The phenomenology of SUSY breaking in this model has been studied in [5]. Neutralino dark matter has not so far been studied in this string model, or any string model involving twisted moduli, although it has been studied in other string models [10]–[13]. However in none of these cases has the question of the naturalness of the predicted dark matter density been addressed and, as discussed, one of the main motivations for the present study is to explore how such results obtained in the non-universal MSSM translate to the case of a "more fundamental" string theory where such non-universality arises automatically. The main motivation for revisiting the model in [4, 5] is that it exhibits non-universal gaugino masses and non-universality between the 3rd family and the 1st and 2nd family squarks and sleptons, which precisely corresponds to the type of non-universality assumed in [1]. This allows a direct comparison between the non-universal MSSM and a corresponding type I string model, since the latter shares many of the dark matter regions previously considered. We will find that dark matter constraints close off much of the parameter space of the type I string model, for example the benchmark points suggested in [5] are either ruled out ($\Omega_{CDM}h^2 \gg \Omega_{CDM}^{WMAP}h^2$) or disfavoured ($\Omega_{CDM}h^2 \ll \Omega_{CDM}^{WMAP}h^2$). However we will find new successful regions of dark matter in the string model, which mirror some of those found in the non-universal MSSM, some of which exhibit degrees of fine-tuning in
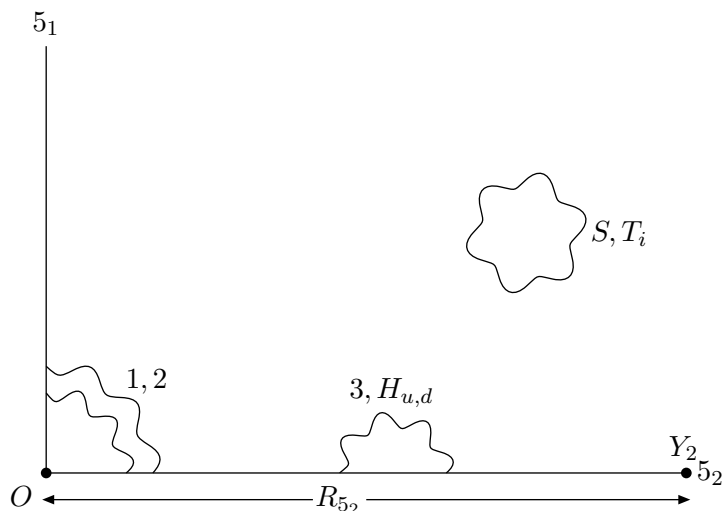
Figure 1: The brane set-up from [4, 5].

agreement with the previous results [1], and some which vary significantly.

The layout of the remainder of this paper is, then, as follows. In section 2 we summarise the string model of [5] and analyse the structures of the GUT scale soft masses specifically with respect to their implications for dark matter. In section 3 we use numerical scans[1] to study the fine-tuning of dark matter within such a model and find important variations from the general results of [1]. In section 4 we present our conclusions.

2. The Model

2.1 The brane set-up

We start with the brane set-up shown in figure 1, originally proposed in [4, 5]. Here we have two perpendicular intersecting D5 branes 5_1 and 5_2. Each holds a copy of the MSSM gauge group. To maintain gauge coupling unification at the GUT scale we take the limit of single brane dominance $R_{5_2} \gg R_{5_1}$. The twisted moduli Y_2 is trapped at a fixed point in the $D5_2$-brane. The untwisted moduli T_i and the dilaton propagate in the 10D bulk. We identify the first and second family scalars with open strings with one end on the 5_1 brane and the other on the 5_2 brane. This localises them at the intersection of the branes and effectively sequesters them from the twisted moduli. The third family scalars and the Higgs bosons are identified with strings on the 5_2 brane.

In such a model the SUSY breaking can come from the twisted moduli (Y_2) localised at a fixed point in the 5_2 brane, the untwisted moduli (T_i) in the bulk or the dilaton (S). Each of these forms of SUSY breaking gives rise to distinct GUT scale soft masses and so to distinct low energy phenomena. As the exact form of their contribution to the SUSY

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[1]As before we use `SOFTSUSY v.1.9.1`[35] to compute the RGE running of the soft parameters and `microMEGAs v.1.3.6`[36] to calculate $\Omega_{CDM}h^2$, δa_μ and $BR(b \to s\gamma)$
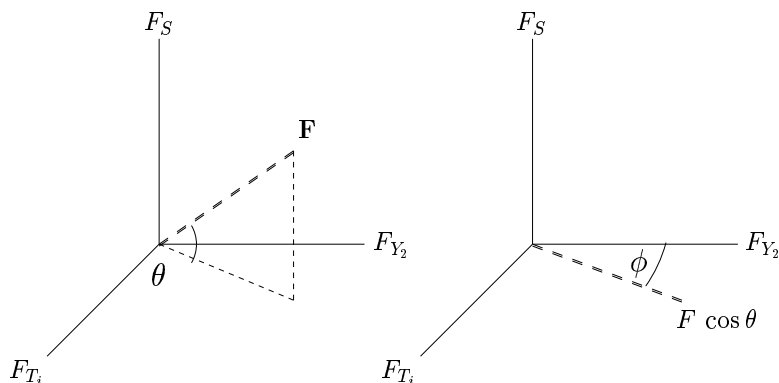
Figure 2: The Goldstino angles are defined to parameterise the F-term SUSY breaking coming from the S,T and Y moduli. $(\theta, \phi) = (0, 0)$ corresponds to twisted moduli (Y_2) SUSY breaking. $(\theta, \phi) = (0, \pi/2)$ corresponds to untwisted moduli (T_i) SUSY breaking. $\theta = \pi/2$ corresponds to dilaton (S) SUSY breaking.

breaking F-terms is not known, we use Goldstino angles [3] to parameterise the relevant contributions of each. These angles are defined as shown in figure 2.

2.2 GUT scale soft masses

The model determines the soft masses at the GUT scale to be [4, 5]:

$$m_{\tilde{Q}}^2, m_{\tilde{L}}^2, m_{\tilde{u}}^2, m_{\tilde{d}}^2, m_{\tilde{e}}^2 = \begin{pmatrix} m_0^2 & 0 & 0 \\ 0 & m_0^2 & 0 \\ 0 & 0 & m_{0,3}^2 \end{pmatrix} \tag{2.1}$$

$$m_{H_u}^2 = m_{H_d}^2 = m_H^2 \tag{2.2}$$

where m_0^2, $m_{0,3}^2$ and m_H^2 are defined:

$$m_0^2 = m_{3/2}^2 \left[1 - \frac{3}{2}\sin^2\theta - \frac{1}{2}\cos^2\theta\,\sin^2\phi - \left(1 - e^{-(T_2+\bar{T}_2)/4}\right)\cos^2\theta\cos^2\phi \right. \tag{2.3}$$

$$-\frac{X}{3}\cos^2\theta\sin^2\phi\,\delta_{GS}\left(1 - e^{-(T_2+\bar{T}_2)/4}\right) + \frac{X^2}{96}\cos^2\theta\sin^2\phi\,e^{-(T_2+\bar{T}_2)/4}(T_2+\bar{T}_2)^2$$

$$\left. -\frac{1}{16\sqrt{3}}\cos^2\theta\cos\phi\sin\phi\,e^{-(T_2+\bar{T}_2)/4}\left\{8(T_2+\bar{T}_2) + \delta_{GS}X\right\}X + \mathcal{O}\left[\frac{\delta_{GS}\,e^{-(T_2+\overline{T_2})/4}}{(T_2+\overline{T_2})}\right] \right]$$

with $X = Y_2 + \overline{Y_2} - \delta_{GS}\ln(T_2 + \overline{T_2})$ where δ_{GS} is the Green-Schwartz parameter.

$$m_{0,3}^2 = m_{3/2}^2\left(1 - \cos^2\theta\,\sin^2\phi\right) \tag{2.4}$$

$$m_H^2 = m_{3/2}^2\left(1 - 3\sin^2\theta\right) \tag{2.5}$$

The soft gaugino masses and trilinears are:

$$M_\alpha = \frac{\sqrt{3}\,m_{3/2}\,g_\alpha^2}{8\pi}\cos\theta\left[\frac{\sin\phi}{\sqrt{3}}\left\{T_2 + \bar{T}_2 + \frac{s_\alpha}{4\pi}\delta_{GS}\right\}\right] \tag{2.6}$$

$$- \cos\phi \left\{ \frac{\delta_{GS}}{T_2 + \overline{T}_2} - \frac{s_\alpha}{4\pi} \right\} + \mathcal{O} \left[\left(\frac{\delta_{GS}}{T_2 + \overline{T}_2} \right)^2 \right] \right]$$

$$\mathcal{A} = -m_{3/2} \left(\cos\theta \, \sin\phi + \mathcal{O} \left[\frac{\delta_{GS}}{(T_2 + \overline{T}_2)^2} \right] \right) \tag{2.7}$$

where we follow [14] in taking the parameter s_α to be equal to the MSSM 1-loop β-function coefficients: $s_\alpha = \beta_\alpha$ where $\beta_\alpha = 2\pi \{33/5, 1, -3\}$. Note that all the soft masses scale as $m_{3/2}$ as expected in any SUGRA theory.

2.3 Fine-tuning and the set of input parameters

The measure we use to study the fine-tuning required to provide electroweak symmetry breaking is [15]-[29][2]:

$$\Delta_a^{\text{EW}} = \frac{\partial \ln \left(m_Z^2 \right)}{\partial \ln (a)} \tag{2.8}$$

Similarly the measure we use to study the fine-tuning of dark matter is the sensitivity parameter [1, 30, 31]:

$$\Delta_a^\Omega = \frac{\partial \ln \left(\Omega_{CDM} h^2 \right)}{\partial \ln (a)} \tag{2.9}$$

Clearly the value of Δ^Ω depends directly on our choice of inputs for a theory. In the non-universal MSSM studied previously we took our inputs at the high energy (GUT) scale as $a = a_{MSSM}$ where:

$$a_{MSSM} \in \{m_0, m_{0,3}, M_1, M_2, M_3, A_0, \tan\beta, \text{sign}(\mu)\} \tag{2.10}$$

Here m_0 is the soft scalar mass of the first and second family of squarks and sleptons, $m_{0,3}$ is the soft scalar mass of the third family of squarks and sleptons and Higgs doublets, M_i are the three soft gaugino masses, A_0 is the universal trilinear soft mass parameter, $\tan\beta$ is the ratio of Higgs vacuum expectation values, and μ is the Higgsino mass parameter.

Within the present type I string model we take $a = a_{string}$ where:

$$a_{string} \in \{m_{3/2}, \delta_{GS}, T_2 + \overline{T}_2, Y_2 + \overline{Y}_2, \theta, \phi, \tan\beta, \text{sign}(\mu)\} \tag{2.11}$$

Here $\tan\beta$ and $\text{sign}(\mu)$ are as in the general MSSM study as they result from the requirement that the model provide radiative electroweak symmetry breaking. θ and ϕ are the Goldstino angles that parameterise the different contributions to SUSY breaking from the moduli and the dilaton. The remaining parameters are directly related to the moduli. The untwisted moduli T_i determine the radii of compactification. $T_2 + \overline{T}_2$ parameterises the compactification radius in the 5_2 direction via the relation[3]:

$$R_{5_2} = \frac{1}{2} \sqrt{T_2 + \overline{T}_2} \tag{2.12}$$

As the twisted moduli are trapped at the fixed point at one end of the brane and the 1st and 2nd families of scalars are trapped at the other end of the brane, the radius of

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[2]See [1] for a discussion of the use of these sensitivity parameters to measure fine-tuning.

Region	ϕ	$M_1 : M_2 : M_3$
Twisted moduli (Y_2) dominated	0	$3.5 : 0.7 : -1.3$
Untwisted moduli (T_i) dominated	$\pi/2$	$5.7 : 26 : 38$

Table 1: The ratio of the GUT scale gaugino masses in the twisted moduli (Y_2) and untwisted moduli (T_i) SUSY breaking limits.

compactification, and therefore $T_2 + \overline{T}_2$, governs the degree of sequestering. This is evident in the limits of eq. (2.3): as $T_2 + \overline{T}_2 \to \infty$, $m_0^2 \to 0$.

Within this paper we follow [5] in taking $T_2 + \overline{T}_2 = 50$ and $Y_2 + \overline{Y}_2 = 0$. This maintains the validity of the series expansion in $\delta_{GS}/(T_2 + \overline{T}_2)$ used to determine the F-terms. However, as these VeVs are essentially arbitrary, we include them in our set of parameters for determining dark matter fine-tuning.

δ_{GS} is a model dependent parameter that depends upon the details of the anomaly cancellation in the twisted sector. This calculation is beyond the scope of this paper and we set $\delta_{GS} = -10$ throughout. However this value can vary and so we include it in our calculation of fine-tuning parameters.

2.4 The structure of the neutralino

The principle factors in the determination of the dark matter relic density are the mass and composition of the lightest neutralino. This is determined by the ratio between M_1, M_2 and μ at the low energy scale. Though we cannot predict the size of μ from the form of the soft masses, we can find M_1 and M_2. The values of M_i at m_{GUT} can be simplified from eq. (2.6) once we have set $T_2 + \overline{T}_2$ and δ_{GS}:

$$
\begin{aligned}
M_1 &= 0.03 m_{3/2} \cos\theta \, (5.7 \sin\phi + 3.5 \cos\phi) \\
M_2 &= 0.03 m_{3/2} \cos\theta \, (26 \sin\phi + 0.7 \cos\phi) \\
M_3 &= 0.03 m_{3/2} \cos\theta \, (38 \sin\phi - 1.3 \cos\phi)
\end{aligned}
\tag{2.13}
$$

The overall magnitude of the gaugino masses is set by $m_{3/2}$ and $\cos\theta$. The ratio of GUT scale gaugino masses is determined by ϕ, as shown in table 1. To analyse the low energy gaugino mass ratio, and so study the composition of the $\tilde{\chi}_1^0$, we can use the rule of thumb[3] that $M_1(M_{SUSY}) \approx 0.4 M_1(m_{GUT})$ and $M_2(M_{SUSY}) \approx 0.8 M_2(m_{GUT})$. This allows us to see that in the twisted moduli dominated limit, in the absence of small μ, we have Wino dark matter. In the untwisted moduli dominated limit, again without small μ, we have Bino dark matter. To find the Wino/Bino well-tempered region we need to find the value of ϕ that gives $M_1(m_{SUSY}) \approx M_2(m_{SUSY})$. This occurs when $M_1(m_{GUT}) \approx 2 M_2(m_{GUT})$ and so the switch from Bino to Wino dark matter will occur around $\phi \approx 0.05$. Therefore to study Wino/Bino "well-tempered" dark matter we should consider low values of ϕ. At

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[3] The exact relation between the GUT scale and low energy masses is determined by the RGEs. We can use this simple rule of thumb in the case of the gauginos because their one-loop RGEs are straightforward, for their explicit form see [32]

Point	θ	ϕ	$m_{3/2}$(TeV)	$\tan\beta$	$\tilde{\chi}_1^0$	$\Omega_{CDM}h^2$
A	0	0	5	4	Wino	$\Omega_{CDM}h^2 \ll \Omega_{CDM}^{WMAP}h^2$
B	0.1	0.1	2	10	Bino	$\Omega_{CDM}h^2 \gg \Omega_{CDM}^{WMAP}h^2$
C	0.6	0.1	2	20	Bino	$\Omega_{CDM}h^2 \gg \Omega_{CDM}^{WMAP}h^2$

Table 2: Benchmark points from [5]. B and C overclose the universe and so are ruled out by dark matter. A lies in a region inaccessible within our studies as the parameter space has disappeared for $m_t = 172.7$ GeV. However even if the parameter space were allowed, the LSP would be Wino and so could not reproduce the observed dark matter density.

lower values of ϕ dark matter will be Wino and so will annihilate too efficiently to explain the observed dark matter. At larger ϕ, dark matter will be Bino or Bino/Higgsino.

In table 1 we have not included the dilaton dominated limit $\theta = \pi/2$ for two reasons. Firstly, as $\theta \to \pi/2$, $M_i \to 0$ and the parameter space will be ruled out by LEP bounds on the neutralinos, charginos and the gluino. As $\cos\theta$ is a common coefficient, the degree of dilaton contribution only affects the overall mass scale of the gauginos, not their composition. Secondly we are forbidden from accessing $\theta = \pi/2$, the dilaton dominated limit, by eq. (2.5). Within this paper we keep the squared Higgs mass positive at the GUT scale and so limit our studies to $\theta < sin^{-1}\left(1/\sqrt{3}\right)$. Therefore the dilaton contribution can only suppress the gaugino masses by a factor of 0.8 at the most. The primary effect of θ on the phenomenology is through the sfermion and Higgs masses.

By considering the structures of the neutralino masses we can quickly analyse the implications of dark matter for the benchmark points proposed in [5]. In table 2 we list the soft parameters that define the three benchmark points and note the resulting composition of the LSP. Point A corresponds to the twisted moduli dominated limit and the LSP is Wino. Wino dark matter annihilates efficiently in the early universe resulting in a relic density far lower than that observed today. For point A to remain valid, there would have to be non-thermal production of SUSY dark matter or some other, non-SUSY, particle responsible for the observed relic density[4].

Points B and C both result in Bino dark matter. In general Bino dark matter does not annihilate efficiently, often resulting in a relic density much greater than that observed. For the density to be in agreement with the measured density, certain annihilation channels need to be enhanced. This can happen if (i) the NLSP is close in mass to the neutralino, allowing for coannihilation, (ii) neutralinos can annihilate to a real on-shell Higgs or Z or (iii) there exist light sfermions that can mediate neutralino annihilation via t-channel sfermion exchange. None of these mechanisms exist in the case of points B or C, resulting in a predicted dark matter density far in excess of that measured by WMAP.

As the previously proposed benchmark points fail, we go on to scan the parameter space to find points that agree with the WMAP measurement of $\Omega_{CDM}h^2$.

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[4]As we will show in section 3.1, this point is also ruled out by LEP bounds on the lightest Higgs if we take $m_t = 172.7$ GeV, as we do throughout this paper.

Soft Mass	Value
m_0	$3.7 \times 10^{-6} \ m_{3/2}$
$m_{0,3}$	$m_{3/2}$
m_H	$m_{3/2}$
M_1	$0.1 \ m_{3/2}$
M_2	$0.02 \ m_{3/2}$
M_3	$-0.04 \ m_{3/2}$
A	0

Table 3: In the twisted moduli (Y_2) dominated limit, $\theta = \phi = 0$, the soft masses take the form shown. This limit is characterised by the exponential suppression of the 1st and 2nd family scalar soft masses and a light Wino LSP.

3. Results

3.1 Twisted moduli dominated SUSY breaking

In the twisted moduli dominated limit ($\theta = \phi = 0$) the soft masses simplify to the values shown in table 3. In this regime the 1st and 2nd family scalars have exponentially suppressed soft masses due to their sequestering from the twisted moduli. The third family scalars and the Higgs bosons have a universal soft mass equal to $m_{3/2}$. Finally the lightest neutralino is Wino and very light.

In figures 3(a)–(d) we examine the phenomenology of the parameter space as T-moduli contributions are gradually switched on by slowly increasing ϕ from 0. In the twisted moduli dominated limit (figure 3(a)) the parameter space is either closed off by LEP bounds on the lightest Higgs and chargino or because $\mu^2 < 0$, resulting in a failure of radiative electroweak symmetry breaking. This disagrees with [5] because we take $m_t = 172.7$ GeV as opposed to $m_t = 178$ GeV. Therefore the twisted moduli dominated limit is ruled out by experimental bounds for the present top mass.

In figure 3(b)-(d) we take incrementally larger values of $\phi = 0.05$, 0.07 and 0.1 respectively. This has three primary effects. Firstly M_2 increases, and to a lesser extent so does M_1 from eq. (2.13). This changes the LSP from Wino to Bino and quickly increases the mass of the charginos, helping to satisfy LEP bounds. Secondly the 1st and 2nd family soft scalar masses receive a substantial contribution from the T-moduli from eq. (2.3). Finally M_3 becomes positive and then steadily increases in size, helping to mitigate the bounds from REWSB and from the LEP bounds on the lightest Higgs boson.

The combination of these effects opens up the parameter space as we increase ϕ, where the area of parameter space consistent with collider phenomenology is shown as white space in the figures, and within this white space the area consistent with WMAP allowed neutralino dark matter is shown as thin coloured bands, where the colour coding corresponds to the degree of fine-tuning as explained in the figure caption. The first evidence of the model providing a dark matter density in agreement with that measured by WMAP is in figures 3(c) and 3(d). In both of these scans, if μ were large the LSP
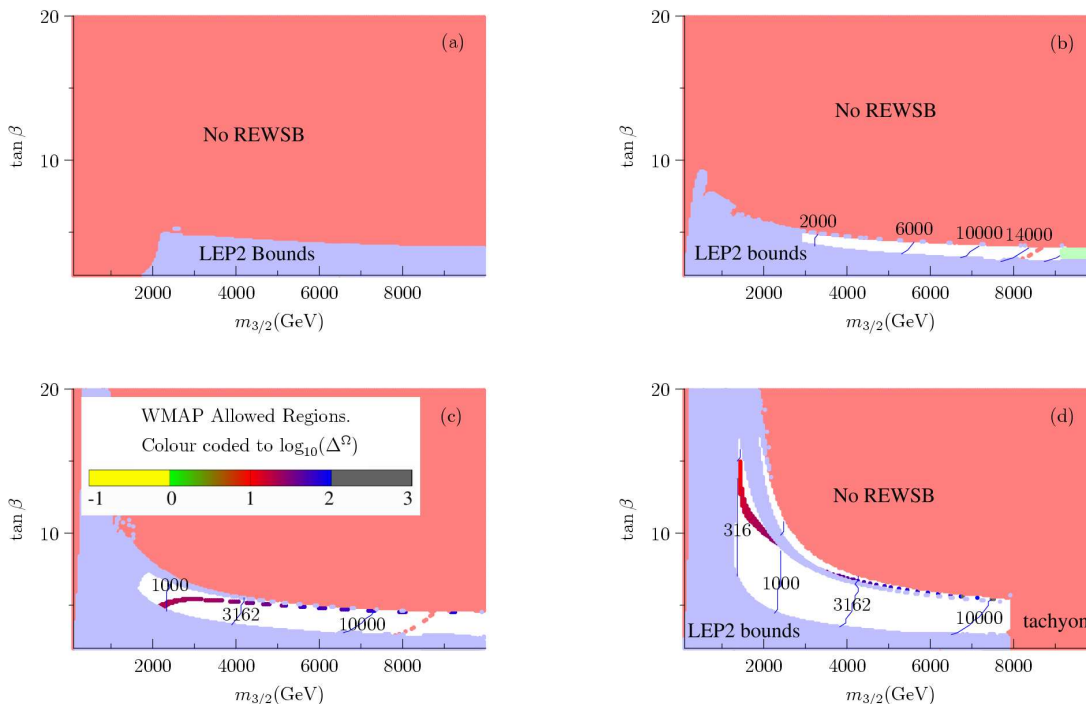
Figure 3: Panel (a) shows the twisted moduli dominated limit $\theta = \phi = 0$. As we switch on contributions from T-moduli, the LEP and REWSB bounds recede. In (b) $\phi = 0.05$, (c) $\phi = 0.07$ and (d) $\phi = 0.1$. $\theta = 0$ throughout. In panels (c) and (d) there are regions allowed by WMAP. These regions are plotted in varying colours corresponding to the degree of fine-tuning they require. In panel (c) we present a legend for this colour coding. Finally, we represent EW tuning by contours in panels (b)-(d). $BR(b \rightarrow s\gamma)$ agrees with measurement at 1σ across the open parameter space but $(g-2)_\mu$ agrees with the Standard Model value. The low energy SUSY spectra corresponding to these panels are discussed in [5]

would be Bino, with a small proportion of Wino. However as much of the parameter space is closed off because $\mu^2 < 0$, along the edge of this region μ will be of a comparable magnitude to M_1 resulting in "well-tempered" Bino/Higgsino dark matter. In such regions, co-annihilation with $\tilde{\chi}_2^0$ and $\tilde{\chi}_1^+$ become significant and reduces the dark matter density to the magnitude observed. However the well-tempered region visible at $4 - 8$TeV is plotted in dark blue, corresponding to a fine-tuning $\Delta^\Omega \approx 60$. This is comparable in magnitude to that of the focus point of the CMSSM. As μ is sensitive to $\tan\beta$ and M_1 is not, there is no reason for these masses to be correlated as is required for Higgsino/Bino dark matter. Therefore it is unsurprising that the tuning is large and the majority of the tuning is due to $\tan\beta$, which strongly affects the calculation of μ.

As we move to lower values of $m_{3/2}$, the colour of the dark matter strip moves from blue to red. This corresponds to a drop in Δ^Ω. To understand this we need to once again consider the composition of the LSP. Away from the region with low μ, the neutralino is primarily Bino with a small but significant Wino component. This results in $\tilde{\chi}_2^0$ and $\tilde{\chi}_1^+$ being slightly heavier than $\tilde{\chi}_1^0$. Across much of the parameter space this mass difference is large enough that co-annihilation effects are unimportant. However, as the overall mass

scale drops, so does the absolute value of the mass difference between the LSP and the NLSPs. Below $m_{3/2} = 4$ TeV, the mass difference is small enough for there to be an appreciable number density of $\tilde{\chi}_1^+$ and $\tilde{\chi}_2^0$ at freeze out to co-annihilate with the LSP. The efficiency of coannihilation is primarily sensitive to the mass difference between the LSP and the NLSP. This mass difference scales slowly with $m_{3/2}$ resulting in a Wino/Bino well-tempered region that exhibits low fine-tuning $\Delta^\Omega \approx 10$, lower than the tuning required for Wino/Bino regions in [1].

In figure 3(b), though there is a region of parameter space that satisfies LEP bounds and REWSB, there is no WMAP allowed strip. This is because here the Wino component of the LSP is already too large and dark matter annihilates too efficiently in the early universe. This is unfortunate as it is only for low ϕ that we have exponentially suppressed soft masses for the 1st and 2nd families. We would like to be able to access such a region of parameter space as light 1st and 2nd family sleptons can provide neutralino annihilation via t-channel slepton exchange. In [1] we found these regions exhibited very low fine-tuning. Such a region is not available in this string model because as soon as we move away from $\phi = 0$ the first and second families gain substantial masses. As soon as we can access Bino dark matter, the sleptons are already too heavy to contribute significantly to neutralino annihilation. Though we fail to find a light slepton bulk region in this limit, in the limit of untwisted moduli dominated SUSY breaking we will find a light $\tilde{\tau}$ bulk region.

Finally we note that the electroweak fine-tuning is large right across this parameter space. This is a direct result of the large values of $m_{3/2}$ that are required to satisfy LEP bounds. When $\phi = 0$, $M_2 = 0.02 m_{3/2}$ from eq. (2.13) and charginos are too light. As we increase ϕ, the coefficient of proportionality between M_2 and $m_{3/2}$ increases but remains small for small ϕ. To reach low $m_{3/2}$ we need to move to regimes in which $\sin \phi \approx \mathcal{O}(1)$, away from the twisted moduli dominated limit. These large values of $m_{3/2}$ are responsible for large electroweak tuning. As $m_{0,3}^2 \approx m_{3/2}^2$, the masses going into our calculation of electroweak symmetry breaking are $\mathcal{O}(m_{3/2})$. We need to tune our soft masses to cancel to provide the correct value of m_Z, orders of magnitude lighter. As we increase $m_{3/2}$ we increase the degree of fine-tuning required. To access regions with low fine-tuning we need to access low $m_{3/2}$, and that means taking large ϕ, as we consider next.

3.2 T-moduli dominated SUSY breaking

In the limit in which all the SUSY breaking comes from the untwisted T-moduli ($\theta = 0$, $\phi = \pi/2$), the soft masses take the form shown in table 4. In the gaugino sector, as $M_1 < M_2$, the lightest neutralino will have no Wino component. Unless there is a part of the parameter space with low μ, the LSP will be Bino. As Bino dark matter on its own generally annihilates extremely inefficiently there would need to be other contributions to the annihilation cross-section to satisfy WMAP bounds. The other defining feature of this limit is that $m_{0,3} = 0$. As the third family particles all pick up masses through loop corrections, they will not be massless at the low energy scale. However these corrections are smallest for $\tilde{\tau}_1$ and will leave it light. This opens up the possibility that t-channel stau exchange and stau co-annihilation will help to suppress the Bino dark matter density.

Soft Mass	Value
m_0	$131\ m_{3/2}$
$m_{0,3}$	0
m_H	$m_{3/2}$
M_1	$0.17\ m_{3/2}$
M_2	$0.78\ m_{3/2}$
M_3	$1.14\ m_{3/2}$
A	$-m_{3/2}$

Table 4: The soft masses in the untwisted moduli (T_i) dominated limit, $\theta = 0, \phi = \pi/2$. This limit is characterised by vanishing 3rd family scalar masses and a Bino LSP.

As the 1st and 2nd family particles have a large soft mass, they will not provide a contribution to the muon $(g-2)$ value. Therefore this limit will not agree with the measured deviation δa_μ from the standard model value[34]. In this limit, the model predicts a value of $(g-2)_\mu$ in agreement with the Standard Model.

In figures 4(a)–(d) we gradually switch on twisted moduli contributions by slowly decreasing ϕ from $\pi/2$ while keeping $\theta = 0$. This immediately gives a non-zero mass to the 3rd family squarks and sleptons. Writing $\phi = \pi/2 - \delta$, for small δ we can write the 3rd family scalar mass:

$$m_{0,3} \approx \frac{\delta}{\sqrt{2}} m_{3/2} \tag{3.1}$$

In figure 4(a) the parameter space of $\tan\beta < 10$ is entirely closed off by LEP bounds on the stau or the stau being the LSP. As we reduce ϕ, we give a soft mass to the stau and so increase its physical mass, helping to satisfy the LEP bound and push its mass above that of the $\tilde{\chi}_1^0$. In figures 4(c),(d) the stau LEP bound is no longer important. The remaining LEP bounds are the Higgs for low $\tan\beta$ and the lightest neutralino for $m_{3/2} < 270$ GeV. Large $\tan\beta$ is ruled out by a failure of REWSB ($\mu^2 < 0$) and the stau being the LSP.

There are 4 distinct regions that satisfy dark matter bounds in the T-moduli dominated limit. Alongside the region in which the stau is the LSP, there is a corresponding dark matter strip in which the stau is close in mass to the neutralino and $\tilde{\chi}_1^0 - \tilde{\tau}$ co-annihilation reduces $\Omega_{CDM}h^2$ to the observed value. This is visible in figures 4(b)–(d) at $m_{3/2} > 450$ GeV. For lower values of $m_{3/2}$, the stau is light enough that $\tilde{\chi}_1^0\tilde{\chi}_1^0 \to \tau^+\tau^-$ via t-channel stau exchange is enhanced to the point that it alone can account for the observed dark matter density. This is the stau analogue of the bulk region found in [1]. As we reduce $m_{3/2}$, we are also reducing the mass of the LSP. Before the LEP bounds close off the parameter space there are regions in which $2m_{\tilde{\chi}_1^0} = m_{Z,h}$. These lie at $m_{3/2} = 310$ GeV and $m_{3/2} = 400$ GeV respectively. In these regions, the lightest neutralino can annihilate via a real on-shell Z or h^0.

Each of these regions has a distinct measure of fine-tuning. The biggest surprise is the stau co-annihilation strip, shown in grey. In contrast to the stau co-annihilation strips studied in [1], this co-annihilation strip exhibits fine-tuning $\Delta^\Omega > 100$. This is an order of
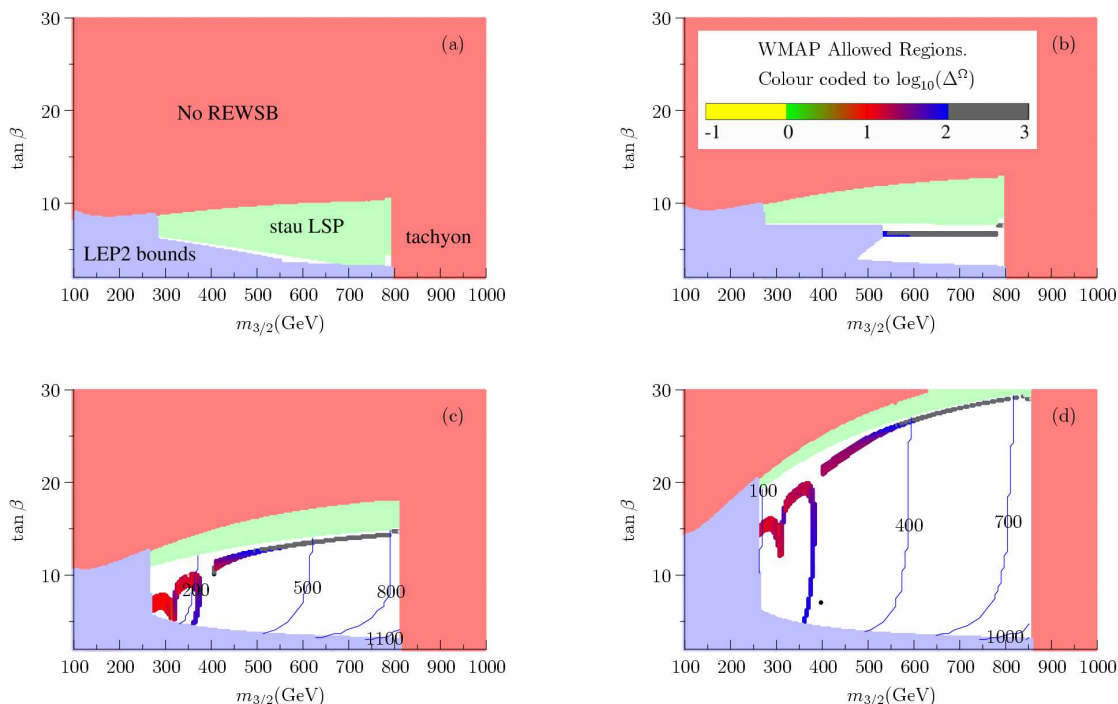
Figure 4: Panel (a) shows the T-moduli dominated limit $\theta = 0, \phi = \pi/2$ in which the parameter space is entirely closed off by experimental bounds. As soon as we move away from $\phi = \pi/2$, the parameter space opens up and we find dark matter allowed regions. In (b) $\phi = 15\pi/32$, (c) $\phi = 7\pi/16$ and (d) $\phi = 3\pi/8$. Once again we switch off the dilaton contributions by taking $\theta = 0$ throughout. In panel (a) we label the different bounds that rule out the parameter space. This colour coding holds true for all the plots. In panel (b)-(d) the WMAP allowed regions are plotted in varying colours. The legend in panel (b) links the colour to the degree of fine-tuning. EW fine-tuning is represented by contours in panels (c) and (d). $BR(b \to s\gamma)$ agrees with measurement at 1σ across the open parameter space but $(g - 2)_\mu$ agrees with the Standard Model value. The SUSY spectra corresponding to these panels are discussed in [5].

magnitude increase over previous stau co-annihilation regions. The reason for this is the extreme sensitivity to ϕ highlighted by eq. (3.1). In previous studies the soft stau mass was so light that loop corrections from the gauginos dominated the determination of its low energy mass. This reduced the sensitivity to variations in the soft stau mass and resulted in the low energy stau and neutralino masses being correlated. In this model, the extreme sensitivity of the stau soft mass to ϕ (for $\phi = 1.47$, a 10% variation in ϕ results in a 150% change in $m_{0,3}$) breaks this correspondence. As a result, for $\theta = 0$, the model does not have a region in which $m_{\tilde{\tau}}$ and $m_{\tilde{\chi}_1^0}$ are correlated.

We can see this by considering the effect of changing from varying the soft mass directly to varying it via ϕ. Under a change of variables:

$$\Delta_\phi^\Omega = \sum_{a_{MSSM}} \frac{\phi}{a_{MSSM}} \frac{\partial a_{MSSM}}{\partial \phi} \Delta_{a_{MSSM}}^\Omega \tag{3.2}$$

When $\theta = 0$, the coefficient of proportionality between Δ_ϕ^Ω and $\Delta_{m_{0,3}}^\Omega$ is $\phi \tan \phi$, so as $\phi \to \pi/2$, $\Delta^\Omega \to \infty$. This dramatically demonstrates the model dependence of fine-tuning.

Soft Mass	Value
m_0^2	$-0.5\ m_{3/2}^2$
$m_{0,3}^2$	$m_{3/2}^2$
m_H^2	$-2\ m_{3/2}^2$
M_i	0
A	0

Table 5: The soft masses in the dilaton (S) dominated limit, $\theta = \pi/2$. This limit is characterised by vanishing gaugino masses and negative Higgs (mass)^2.

Eq. (3.2) is exact and a similar change of variables can be performed to find all of the $\Delta^\Omega_{a_{string}}$ in terms of $\Delta^\Omega_{a_{MSSM}}$. In general these expressions are large and not particularly informative. However in cases such as that of the $\tilde{\tau}$ coannihilation region, we can use eq. (3.2) to understand the change in the fine-tuning.

The bulk region is shown in red in figures 4(c),(d) corresponding to Δ^Ω of order 10. This tuning is entirely from ϕ. In [1], the tuning of the bulk region came equally from $\Delta^\Omega_{M_1}$ and $\Delta^\Omega_{m_0}$ where m_0 was the soft mass of the slepton that mediated t-channel annihilation. In [1] the total tuning of the bulk region was found to be low, $\Delta^\Omega \approx 1$. When we change variables from a_{MSSM} to a_{string}, for $\delta \approx 0.1$, $\theta = 0$, eq. (3.2) gives $\Delta^\Omega_\phi \approx 10\Delta^\Omega_{m_{0,3}}$ in the bulk region. This explains the order of magnitude increase in the tuning.

Finally we consider the resonances. The lower edge of the Higgs resonance exhibits a tuning $\Delta^\Omega \approx 50$ whereas the edge at larger $m_{3/2}$ is so steep that the scan has failed to resolve it. What we can see of it exhibits tuning well in excess of 100. In contrast the Z resonance exhibits relatively low fine-tuning. This is because annihilation via an s-channel Z is inefficient and provides only a small contribution to the total annihilation cross-section. This is because the Z is spin 1, whereas the neutralino is a spin 1/2 Majorana fermion. This means that in the $v_{\tilde{\chi}_1^0} \to 0$ limit, the annihilation cross-section via on-shell Z production becomes negligible. As this contribution is small, it hardly affects the dark matter fine-tuning.

The electroweak fine-tuning is shown by contours on the open parameter space. As we noted in the previous section, electroweak fine-tuning depends closely on the largest 3rd family masses. As we can access low $m_{3/2}$ for large ϕ, we end up with electroweak fine-tuning $\mathcal{O}(100)$, similar to the lowest electroweak fine-tuning found in the MSSM.

3.3 Switching on the dilaton.

In the limit of dilaton dominated SUSY breaking, $\theta = \pi/2$ the soft mass terms take the form shown in table 5. This structure of soft masses gives rise to a plethora of problems. Firstly, negative soft sfermion mass squareds will result in tachyons. Secondly massless gauginos are ruled out by LEP. However the biggest problem lies in the Higgs sector. If the soft term m_H^2 is negative we run the risk of breaking electroweak symmetry at the GUT scale. This happens when $m_H^2 + \mu^2 < 0$ at the GUT scale. We steer clear of such
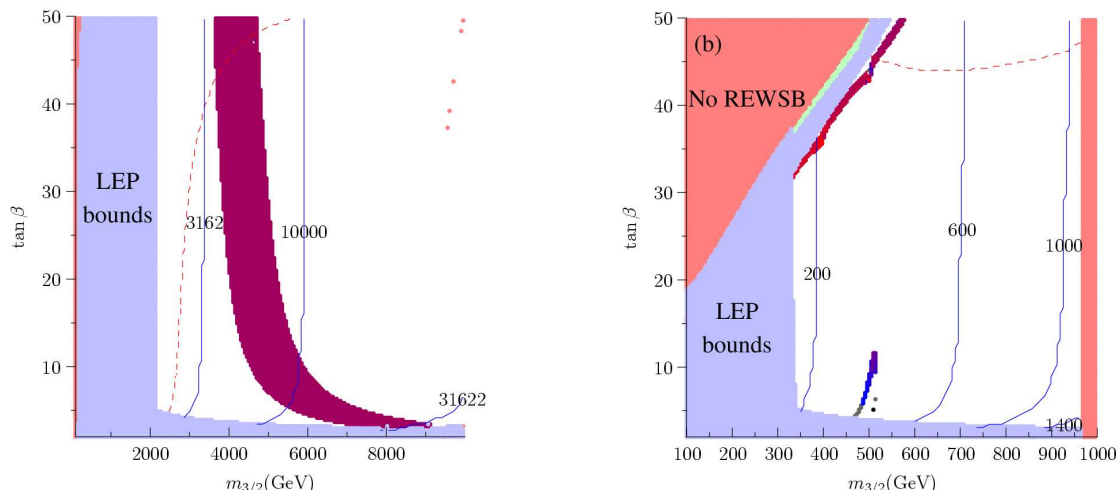
Figure 5: Here we show the maximum dilaton contribution $\theta = 0.6$. For larger values of θ, $m^2_{H_{1,2}} < 0$ at the GUT scale. The regions that satisfy dark matter constraints are plotted in varying colours to represent the required quantity of fine-tuning. This colour coding is as per the legend in figure 4(b). The electroweak fine-tuning is represented by contours in the open parameter space. The $BR(b \to s\gamma)$ 1σ limit is plotted as a red dashed line. In panel (a) $\phi = 0.06$, here we have maximally tempered Bino/Wino/Higgsino dark matter, plotted in purple. In panel (b) $\phi = \pi/2$, the limit in which there is no twisted moduli (Y_2) contribution. Again $(g-2)_\mu$ agrees with the Standard Model.

regions by constraining our parameters to give $m^2_H > 0$. This allows us to impose the limit $0 < \theta < 0.6$.

When we consider the maximum allowed dilaton contribution, there are two interesting limits. For $(\theta = 0.6, \phi = 0)$ we have (S, Y_2) SUSY breaking. When $(\theta = 0.6, \phi = \pi/2)$ we have (S, T_i) SUSY breaking.

For $\phi = 0$, dark matter is still Wino and so cannot reproduce the observed dark matter density. The only change is that we can access large values of $\tan\beta$. Therefore we cannot have a model in which there is no T-moduli contribution to SUSY breaking and reproduce the observed dark matter density.

In figure 5(b) $\phi = \pi/2$, $\theta = 0.6$ giving $M_1 < M_2$ and hence the LSP is Bino. By introducing non-zero θ we increases the stau mass and avoid the LEP bounds on the stau that ruled out $\theta = 0$, $\phi = \pi/2$. It is only for large $\tan\beta$ that the stau is light enough to contribute to neutralino annihilation via t-channel $\tilde{\tau}$ exchange. As before this region is shown in red, corresponding to $\Delta^\Omega \approx 10$. As we can still access low $m_{3/2}$, there exists a region in which the neutralinos can annihilate via the production of a real on-shell h^0 or Z. The Z resonance shows up as a small blip in the bulk region at $m_{3/2} = 400$ GeV. The h^0 resonance appears as a highly tuned region (dark blue) in the stau bulk region around $m_{3/2} = 500$ GeV and also at $\tan\beta = 5-10$. For $\tan\beta = 10-40$, even resonant annihilation via on-shell Higgs production is not enough to suppress the dark matter density.

As we steadily decrease ϕ, the staus increase in mass removing the stau bulk region. Small ϕ also reduces the gaugino masses, requiring ever larger values of $m_{3/2}$ to satisfy LEP bounds. There is no change in the dark matter phenomenology until $\phi = 0.06$, when

Dark Matter Region	θ	ϕ	$m_{3/2}$(TeV)	Typical Δ^{Ω}	Typical Δ^{EW}
Higgsino/Bino	0-0.6	< 0.4	1-10	60	> 3000
Wino/Bino	0-0.6	≈ 0.06	1-3	10	$300 - 3000$
Bino/Wino/Higgsino	0-0.6	≈ 0.06	2-5	10-20	1000-6000
$\tilde{\tau}$-co-annihilation	0-0.6	> 0.8	0.4-0.9	100	500-800
t-channel $\tilde{\tau}$ exchange	0-0.6	> 0.8	0.25-0.45	10	100-200
h^0 resonance	0-0.6	> 0.4	≈ 0.4	> 80	200
Z^0 resonance	< 0.3	> 0.4	≈ 0.3	4-20	130

Table 6: A summary of the successful regions of parameter space in the type I string model considered here that satisfy experimental bounds on the dark matter density with corresponding typical values of Δ^{Ω} and Δ^{EW}.

the neutralino acquires a large Wino component. In figure 5(a) we display this region of parameter space. Here $M_1 \approx M_2 \approx \mu$ at the low energy scale, resulting in maximally tempered Bino/Wino/Higgsino dark matter as proposed in [1]. This in turn gives a wide dark matter annihilation strip shown in purple that corresponds to $\Delta^{\Omega} = 23$. This tuning arises from the soft mass sensitivity to ϕ. This dependence is understandable as it is ϕ that determines the size of the Bino and Wino contributions to the lightest neutralino.

The electroweak fine-tuning is dependent upon the size of $m_{3/2}$. Therefore figure 5(b) exhibits low Δ^{EW} in agreement with figure 4 and figure 5(a) exhibits large Δ^{EW} as in figure 3.

4. Conclusions

We have used the measured dark matter relic density to constrain a semi-realistic type I string model. In the model considered supersymmetry breaking arises from F-terms of moduli fields parameterised in terms of Goldstino angles, which automatically gives rise to non-universal soft third sfamily and gaugino masses, which precisely corresponds to the type of non-universality assumed in the MSSM [1]. We have studied fine-tuning in the string model for both electroweak symmetry breaking and dark matter. We have found that dark matter constraints close off much of the parameter space of the type I string model, for example the benchmark points suggested in [5] are either ruled out $(\Omega_{CDM}h^2 \gg \Omega_{CDM}^{WMAP}h^2)$ or disfavoured $(\Omega_{CDM}h^2 \ll \Omega_{CDM}^{WMAP}h^2)$. However, by performing a comprehensive scan over the parameter space, we found successful regions of dark matter within the string model. Some of these mirror regions found in the non-universal MSSM studies in [1]. When we consider fine-tuning, some regions exhibit degrees of fine-tuning in agreement with the previous results while others vary significantly. The results are summarised in table 6.

From table 6 it can be seen that the observed dark matter density tightly constrains the available parameter space. For $\phi > 0.07$, without unusual contributions to the annihilation cross-section the model predicts an overabundance of dark matter that would over close the universe. Equally for $\phi < 0.05$, the LSP is Wino and the model predicts a dark

matter abundance orders of magnitude less than that observed. By imposing dark matter constraints we have ruled out the benchmark points proposed in [1]. Instead, we propose a benchmark point within the region of lowest fine-tuning, the stau bulk region combined with on-shell Z production. The SUSY spectrum of this point is presented in table 7.

In addition to constraining our models, we have been able to study how fine tuning varies between the MSSM studied in [1] and a type I string model of SUSY breaking, which was one of our main motivations for this study. From table 6 it can be seen that, in the string model, the lowest dark matter fine-tuning exists in the bulk region, corresponding to t-channel $\tilde{\tau}$ exchange. The Z resonance, the well tempered Bino/Wino and the maximally tempered Bino/Wino/Higgsino regions also have low dark matter fine-tuning. Of these, the lowest electroweak fine-tuning arises in the bulk (t-channel $\tilde{\tau}$ exchange) and Z resonance regions. These results are consistent with the conclusions based on the previous MSSM analysis, although the bulk region in the MSSM corresponding to first and second family slepton exchange cannot be accessed in the string model as discussed. Thus in most cases the degree of fine-tuning is found to be the same order of magnitude as found for similar dark matter regions within the MSSM. However this is not always the case. Whereas the well tempered Higgsino/Bino region in table 6 continues to be highly fine-tuned as in the MSSM, the well tempered Bino/Wino in table 6 has a fine tuning of about 10 as compared to the MSSM value of about 30, making this scenario more natural in the framework of string theories such as the one considered here.

In some cases there is a sharp disagreement between the fine tuning calculated in the MSSM and in the string model, for example in the stau co-annihilation region. Due to the form of the SUSY breaking in this model, the stau mass, and so the dark matter density, is very sensitive to ϕ which leads to an order of magnitude increase in the dark matter fine-tuning in the string model as compared to the MSSM, making this region less natural in the string model. This can be understood via eq. (3.2) which shows that, through a general change of variables, the variation of the fine-tuning between a general MSSM model and a string model can be calculated. In principle a similar change of variables is responsible for the all the differences in fine tuning calculated in the MSSM and the string model. In practice however, such a change of variables is not analytically tractable, and numerical methods such as those used in the present paper are required in order to obtain quantitative results. However the results in this paper indicate a general strategy for reducing fine tuning within string models, namely to search for string models that minimise the coefficients of the tuning measures. This in turn will minimise Δ^{Ω}, providing more natural dark matter than the MSSM for a given region of parameter space. Such a strategy could also be employed to reduce electroweak fine tuning once the solution to the μ problem is properly understood within the framework of string theory.

Finally we note that in this paper we have not included the effect of string loop corrections in estimating the degree of fine-tuning, although we have included the effect of varying δ_{GS} which parametrizes the one-loop Green-Schwartz anomaly. The magnitude of string loop corrections is in general difficult to quantify, although there has been some recent progress in this area [37] which indicates that at least in certain constructions the magnitude of such corrections could be suppressed by a factor of $1/256\pi^6$. In general the

study of such corrections, which in principle could play a role in the results presented in this paper, is beyond the scope of the present paper.

Acknowledgments

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Point	A'
θ	0
ϕ	$3\pi/8$
$m_{3/2}$	310
$\tan\beta$	13
m_{h^0}	115
m_{A^0}	550
m_{H^0}	550
$m_{H^\pm}$	556
$m_{\tilde\chi_1^0}$	44.5
$m_{\tilde\chi_2^0}$	213
$m_{\tilde\chi_1^\pm}$	213
$m_{\tilde g}$	930
$m_{\tilde t_1}$	546
$m_{\tilde t_2}$	757
$m_{\tilde c_L}, m_{\tilde u_L}$	3390
$m_{\tilde c_R}, m_{\tilde u_R}$	3390
$m_{\tilde b_1}$	687
$m_{\tilde b_2}$	739
$m_{\tilde s_L}, m_{\tilde d_L}$	3390
$m_{\tilde s_R}, m_{\tilde d_R}$	3390
$m_{\tilde\tau_1}$	104
$m_{\tilde\tau_2}$	222
$m_{\tilde\mu_L}, m_{\tilde e_L}$	3290
$m_{\tilde\mu_2}, m_{\tilde e_2}$	3280
$m_{\tilde\nu_e}, m_{\tilde\nu_\mu}$	3290
$m_{\tilde\nu_\tau}$	197
LSP	$\tilde\chi_1^0$

Table 7: Sample spectra for benchmark point A' corresponding to a point in figure 4(d) at $m_{3/2} = 310$ GeV and $\tan\beta = 13$. At this point we satisfy WMAP bounds on the dark matter density, $BR(b \to s\gamma)$ and all present mass bounds. This point requires a tuning to achieve electroweak symmetry breaking: $\Delta^{EW} = 125$, and a tuning to agree with WMAP: $\Delta^\Omega = 3.9$. The annihilation of neutralinos in the early universe is due to 40% $\tilde\chi_1^0\tilde\chi_1^0 \to \tau^+\tau^-$ via t-channel $\tilde\tau$ exchange and 60% $\tilde\chi_1^0\tilde\chi_1^0 \to f\overline{f}$ via the production of an on-shell Z. All masses are in GeV.